Computer Simulation for the Optimal, Low-thrust LEO-TO-MOLNIYA Transfer Using NASA'S SEPSPOT Program

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ABSTRACT

The nature of this work consists of running a computer simulation using NASA's SEPSPOT program to solve for the optimal low-thrust Earth-orbit trajectory for the LEO-to-Molniya transfer. For this scenario, a spacecraft is transferred from low Earth orbit to the final mission orbit by using various initial thrust accelerations ranging from 10^{-1} to 10^{-2} g. Furthermore, the numerical solutions obtained from the program for all the different cases are validated by comparing them with the analytic solutions derived from analytical blended control methods.

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Chapter 1

INTRODUCTION

1.1 Historical Perspective

During the past few decades, the aerospace industry has attempted to develop a solar-electric-propulsion planetary orbiter spacecraft, since such a vehicle would have a significantly increased propellant efficiency, greater maneuverability, larger payload capabilities, and a greater lifetime than a conventional chemical-propulsion space vehicle [1]. In the fall of 1998, NASA's New Millennium Program launched Deep Space 1, an ion propelled spacecraft, on an eleven month mission; however, the space vehicle exceeded NASA's expectations and kept running for a few more years, conducting testing on its ion engine [2].

Apart from successfully completing its primary mission, Deep Space 1 also flew by the comet Borrelly and transmitted the best close-up pictures and best scientific data ever gathered from a comet. Despite the fact Deep Space 1 was retired in December 2001, its spare ion engine has been running continuously at the Jet Propulsion Laboratory (JPL) in Pasadena, California since October 1998, which demonstrates that the ion engine is an excellent propulsion system for future space exploration missions. From a performance standpoint, the ion engine is capable of delivering ten times as much thrust per kilogram of fuel than conventional chemical engines [2].

Overall, Deep Space 1's spare ion engine ran for over 24,750 hours. To illustrate the effectiveness and efficiency of this engine imagine "if it had been an automobile engine instead of an ion engine, and it was driven for 24,750 hours at 80.5 kilometers per hour (50 mph), it would have traveled 1.93 million kilometers (1.2 million miles) without an oil change or tune up" [2].

1.2 The Problem

An important aspect of ion-propulsion technology is its high exhaust velocity, which allows an ion engine to run on a few hundred grams of propellant per day while allowing the ion-propelled spacecraft to travel faster and farther than any other space vehicle [2]. However, in order to achieve this high exhaust velocity, an ion engine must achieve very high specific impulse values, ranging in a few thousand seconds, along with relatively low-thrust levels in milli-Newtons [1].

As a result of the ion engine's limitations to low-thrust levels, the spacecraft's ion-propulsion systems will be required to operate for extended periods of time during orbit transfers in order to achieve the required mission orbit [1]. Moreover, the extended periods required during orbit transfers, present the significant problem of computing an optimal trajectory while maintaining the spacecraft's solar panels pointed at the sun within tolerance levels; the solar-electric arrays provide power to the ion engine [3].

1.3 Significance of the Study

Many studies have been conducted on this issue in an attempt to find the best method for computing an optimal low-thrust Earth-orbit transfer. The nature of this work is to extend a study by Spencer and Herman [1] in which higher-order collocation methods and analytical blended control methods are applied to solve the optimal trajectory problem.

The contribution made to this study consists on running a computer simulation using NASA's SEPSPOT program on the LEO (low Earth orbit) to Molniya transfer, which was not included in the original project. In addition, the analytical data for the LEO-to-Molniya transfer computed by Spencer [4] is compared with the results obtained from the program.

The solution to this problem is imperative since it is preventing ion-propulsion technology, a very promising technology, from revolutionizing the face of space exploration by reducing orbit transfer through the ion engine's light weight, allowing spacecraft to travel at faster speeds with longer ranges [2], and designing spacecraft with larger payload capabilities and significantly greater lifetimes [1].

An example of how ion propulsion technology has the potential to enhance humanity's space capabilities is the combination of ion propulsion with solar arrays to create a space probe designed to escape from the solar system. Some individuals within the aerospace community predict that by combining these two technologies a spacecraft will be capable of traveling at an ultimate hyperbolic velocity of the order of 200 km/s. This means that such a space probe would be able to exceed by more than a factor of ten the ultimate velocity of Voyager 2, the space probe that did planetary flybys of Jupiter, Saturn, Uranus, and Neptune a few decades ago. Voyager relied on chemical propulsion [5].

In order to create such a spacecraft the solar arrays would have to be of "extremely low weight consisting of a photovoltaic thin film and conductors vapor-deposited on a thin mylar or kapton sheet." In addition, the sheet would have to be relatively small in width and very long in length. On the space probe, this sheet would be turned toward the Sun and stabilized by the ion engines, which would be arranged at the sheet edges [5].

Another possible configuration for the space probe would be for the sheet to be triangular, with the ion engines at the corners. In either case, the thrust vectors would have to be oriented such that a small fraction of the thrust can be used for stretching and stabilizing the sheet; moreover, the ion engines would also be used to unfold the folded sheet initially. However, all these predictions are still theoretical since the current solar arrays do not have the conversion efficiency required; it is predicted that in the near future solar arrays with the necessary conversion efficiency will be designed and manufactured [5].

1.4 Research Questions

This study will attempt to answer the following questions:

1. By doing a comparison of the results computed by SEPSPOT and the analytical data derived by Spencer during the original project, are both results closely related?
2. If the data is not closely related, what factors might have accounted for this?
3. What do the results look like when they are plotted?

Chapter 2

FUNDAMENTAL THEORY

2.1 Previous Work on Low-Thrust Optimization Methods

In past years, the trajectory optimization problem regarding the low-thrust propulsion systems has been investigated in order to find the best solution method. For example, multiple optimal and non-optimal transfer trajectories between specific initial and final orbits have been studied [6]. In addition, a method of averaging that provides a quick trajectory evaluation compared to methods based upon numerical integration of differential equations was developed [7].

Also, in another study, Lawden's "primer vector" theory was used to analyze impulsive and near-impulsive transfers in order to predict the conditions for low-thrust transfers. This study used algebraic approximations to compute the total time and gravity loss for relatively efficient transfers and to demonstrate that gravity losses for a transfer are reduced to a low level if enough burns are done [8].

2.2 Most Recent Work on Low-Thrust Optimization Methods

Herman and Conway [9] found optimal, low-thrust, Earth-moon orbit transfers by applying a method of collocation with nonlinear programming. The Earth orbit of the spacecraft and the final lunar orbit are both arbitrary while the moon is in its actual orbit. Furthermore, the total transfer time is minimized, but the trajectory is also propellant minimizing since the propulsion system operates continuously and prohibits a coast arc.

Also, Herman and Conway discovered that a very low initial thrust acceleration of 10^{-4} g yields flight times of approximately 32 days and requires many revolutions of both the Earth and the moon. In addition, if the problem is solved as two coupled two-body problems by ignoring the third body, then the optimal trajectory is changed slightly. The optimal trajectory is also insensitive to change in the engine specific impulse as long as the same initial thrust acceleration magnitude is used.

On the other hand, Prussing [10] examined minimum-fuel impulsive spacecraft trajectories in which long-duration coast arcs between thrust impulses are possible. If the coast time is long enough that it allows one or more complete revolutions of the central body then the solutions become complicated. This type of scenario presents Lambert's problem in which the determination of the orbit that connects two specified terminal points in a specified time interval brings about multiple solutions; a transfer time long enough to allow N revolutions of the central body has 2N + 1 trajectories that satisfy the boundary value problem.

Lambert's problem is a classical orbit boundary-value problem, which can be thought of as both an orbit determination problem and a spacecraft targeting problem. The solution to this problem in the two-body problem is the conic orbit that connects two specified terminal points in a specified time interval. In order to solve all the trajectories, Prussing developed an algorithm based on the classical Lagrange formulation for an elliptic orbit. Moreover, this procedure is applied to the problem of rendezvous with a target in the same circular orbit as the spacecraft, while the minimum-fuel optimality of the two-impulse trajectory is determined using primer vector theory [10].

Kechichian [11] also studied the minimum-time low-thrust rendezvous and transfer using the epoch mean longitude formulation. The study shows the state and adjoint differential equations as explicit functions of time that include natural orbital elements that stay constant if no perturbations are applied. In addition, the optimal Hamiltonian is time varying while the function that defines the transversality condition at the end time in minimum-time problems is illustrated as constant during the optimal transfer.

Coverstone-Carroll and Williams [12] developed a direct optimization method based on differential inclusion concepts and used the formulation to compute low thrust trajectories. This procedure removes explicit control dependence from the problem statement, which reduces the dimension of the parameter space and requires fewer nonlinear constraints in the resulting nonlinear programming problem. Moreover, the study presents simulations for a two-dimensional gravity-free trajectory, which involves a maximum velocity transfer to a rectilinear path, an Earth-Mars constant specific impulse transfer, an Earth-Jupiter constant specific impulse transfer, and an Earth-Venus-Mars variable specific impulse gravity assist.

In another study, Betts [13] used the direct transcription method, one of the most effective numerical techniques, to solve the trajectory optimization and optimal control problems. This method combines a sparse nonlinear programming algorithm with a discretization of the trajectory dynamics. Furthermore, the vehicle dynamics are defined by using a modified set of equinoctial coordinates while the trajectory modeling is described using these dynamics. Also, in order to demonstrate some special features of this method such as alternate coordinate systems during the transfer and mesh refinement to produce a high fidelity trajectory, the solution for the transfer from Earth to Mars including a swingby of the planet Venus is presented using the direct transcription method.

In addition, Kechichian [14] explored the optimal low-Earth-orbit-Geostationary-Earth-orbit intermediate acceleration orbit transfer by analyzing the problem of minimum-time orbit transfer using intermediate acceleration through precision integration and averaging. In his study, continuous constant accelerations of the order of 10^{-2}g are considered for applications using nuclear propulsion upper stages; in addition, the acceleration vector is optimized in direction with its magnitude held constant throughout the flight. The scenarios examined have trajectories that circle the Earth for only a few orbits before reaching geostationary Earth orbit, and these trajectories have demonstrated to be sensitive to departure and arrival points, requiring the use of the full six-state dynamics for satisfactory and meaningful results. Also, the ΔV losses with respect to very low-acceleration transfers are shown to be small.

2.3 Background on the Current Study

Spencer and Herman [1] focused on optimal low-thrust Earth-orbit transfers using higher-order collocation methods in which several Earth-orbit transfers, the LEO-to-GEO, LEO-to-MEO, and LEO-to-HEO transfers, were computed and then compared to the solutions found through analytical blended control methods. For each of these scenarios, a spacecraft is transferred from LEO to the final mission orbit by using various initial thrust accelerations (TA) ranging from 10^0 to 10^{-2} g. Refer to Figure 1 for the classification of these orbits.

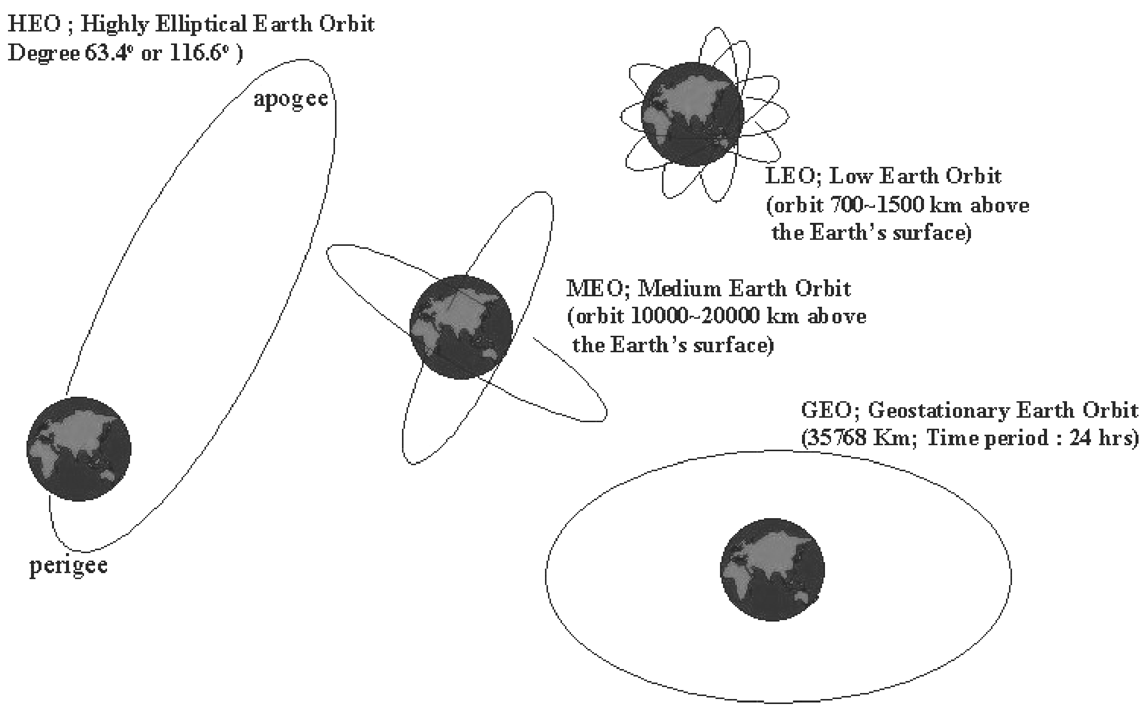

Figure 1: Classification of the GEO, LEO, MEO, and HEO Earth Orbits [15].

Their study involved determining the control time histories of a set of states, a system of first-order ordinary differential equations, from specified initial conditions to the desired final conditions while minimizing a function of the final values of states and/or time. These time histories are determined through a performance function, a scalar function consisting of the values of the states at the final time and the initial and final times, which is minimized while meeting the initial and final conditions of the system of differential equations [1].

2.3.1 Numerical Solution Method

For this method the problem is transformed into a mathematical programming problem (MP) by discretizing the time history solutions and then applying an approximate integration method. By discretizing the time history solutions into L subintervals, not necessarily of equal length, the endpoints of these subintervals are denoted as $\{t_0, t_1, \ldots, t_{i-1}, t_i, t_{i+1}, \ldots, t_{L-1}, t_L\}$. Within a given subinterval $[t_{i-1}, t_i]$ the time history of each solution is approximated by a numerical integration of the system of dynamics [1], which is given by Eq. (2.1) as

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t) \tag{2.1}$$

where the initial conditions for the states are

$$\dot{\bar{x}}(t_I) = \bar{x}_I \tag{2.2}$$

and the desired final conditions are represented by

$$\bar{\Psi}[\bar{x}(t_f)] = 0 \tag{2.3}$$

The original optimal control problem is formulated as a mathematical programming problem (MP) where the controls, $\bar{u}$, are determined to minimize the performance function, $\tilde{J}$, given in Eq. (2.2)

$$\tilde{J} = \phi[\bar{x}(t_F), t_I, t_F] \tag{2.4}$$

Furthermore, in this mathematical programming problem, higher order collocation 7th degree system constraints are applied to solving the system differential equations given in Eq. (2.1), which results in a non-linear programming problem (NLP) where the constraint Jacobian, exhibits a high degree of data sparseness. The software package SNOPT is used to solve the NLP-formulated problem due to its usefulness with sparse matrix problems. This entire method is called DHOC7 [1].

2.3.2 Analytical Solution Method

This method minimizes the propellant usage for a given transfer by assuming that the propellant usage rate is constant during a burn, which means, the burn times for a given maneuver are minimized by maximizing the time rate of change of the particular orbital parameter that governs the burn. During the first burn, thrusting is performed in the orbit plane to increase the apogee radius to the desired circular GEO (geostationary) radius value; in addition, the rate of change of the semimajor axis, da/dt, is maximized by determining the in-plane (α) motion of the thrust direction. Also, during the first burn, the out-of-plane component of the thrust vector does not exist. Afterwards, a coast is initiated and lasts until there is a second burn [1].

For the second burn, the first change from the initial value to the desired final value is the inclination of the spacecraft's orbit, followed by the orbit being circularized to correspond to the GEO. Next, the change in inclination, di/dt, is maximized in order to minimize the burn time. This results in the out-of-plane thrust angle (β) being near ±90 degrees while the inclination change maneuver is centered about the apogee. Figure 2 illustrates the thrust vector and angle definitions [1].

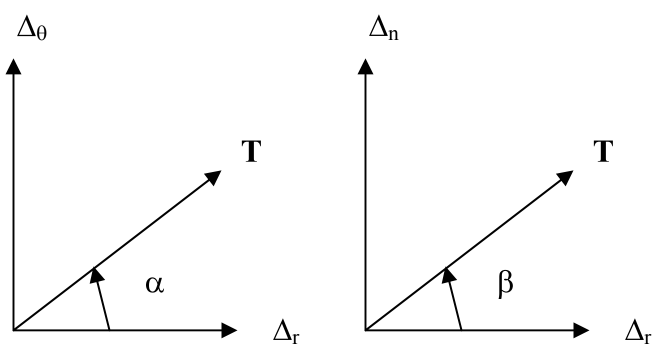

Figure 2: Thrust Vector and Angle Definitions [1]

For various cases of the LEO-to-GEO transfers with a range of TA values, a specific impulse of 1000 sec was used. When a comparison between the analytical solution (Spencer's method) and the numerical solution (DHOC7 method) was done for these cases, the results showed that Spencer's method provides a near-optimal performance by assuming his thrusting strategy. Table 1 denotes the change in total effective velocity between the two solutions [1].

Table 1 Comparison of LEO-to-GEO transfers methods [1]

Effective ΔV, m/s					
Case	**First Burn**	**Second Burn**	**Third Burn**	**Total**	**Change in Total Effective ΔV**
Spencer's results	**3785**	**1237**	**782**	**5804**	**------------**
3 Burns ─ Separate Controls	**3751**	**1244**	**674**	**5668**	**2.40%**
2 Burns ─ Combined Second Burn	**3749**	**1499**	**------------**	**5248**	**10.60%**
2 Burns ─ Optimized Thrust Directions	**3944**	**1133**	**-----------**	**5077**	**14.30%**

Here, the case named "3 Burns-Separate Controls" where the first and third burns are forced to raise the orbit, and the second burn rotates the orbit plane and zeros out the inclination, shows there is a percent error of 2.40% between the numerical and analytical solutions [1].

2.3.3 Equations Used

In order to avoid the singularities that occur in the modified classical orbit elements (a, e, i, Ω, ω, M) when e = 0 and i = 0 deg, modified equinoctial orbit elements must be used to describe the orbit transfers [1]. Therefore, the modified equinoctial orbit elements (p, f, g, h, k, L) must be defined in terms of the modified classical orbital elements as:

$$p = a(1 - e^2) \quad \textbf{(2.5)}$$

$$f = e\cos(\omega + \Omega) \quad \textbf{(2.6)}$$

$$g = e\sin(\omega + \Omega) \quad \textbf{(2.7)}$$

$$h = \tan(i/2)\cos\Omega \quad \textbf{(2.8)}$$

$$k = \tan(i/2)\sin\Omega \quad \textbf{(2.9)}$$

$$L = \Omega + \omega + \upsilon \qquad \textbf{(2.10)}$$

In addition, the equations of motion of a thrusting spacecraft in an inverse square gravity field in terms of the modified equinoctial orbit elements are:

$$\dot{p} = (2p/\omega)(p/\mu)^{1/2}\Delta_\theta \qquad \textbf{(2.11)}$$

$$\dot{f} = (p/\mu)^{1/2}\{\Delta_r \sin L + [(\omega+1)\cos L + f](\Delta_\theta/\omega) - (h\sin L - k\cos L)(g\Delta_h/\omega)\} \qquad \textbf{(2.12)}$$

$$\dot{g} = (p/\mu)^{1/2}\{-\Delta_r \cos L + [(\omega+1)\sin L + g](\Delta_\theta/\omega) + (h\sin L - k\cos L)(f\Delta_h/\omega)\} \qquad \textbf{(2.13)}$$

$$\dot{h} = (p/\mu)^{1/2}(s^2\Delta_h/2\omega)\cos L \qquad \textbf{(2.14)}$$

$$\dot{k} = (p/\mu)^{1/2}(s^2\Delta_h/2\omega)\sin L \qquad \textbf{(2.15)}$$

$$\dot{L} = (\mu p)^{1/2}(\omega/p)^2 + 1/\omega(p/\mu)^{1/2}(h\sin L - k\cos L)\Delta_h \qquad \textbf{(2.16)}$$

$$\dot{m} = -T/c \qquad \textbf{(2.17)}$$

$$\dot{\eta} = -(T/m_0)(1/c) \qquad \textbf{(2.18)}$$

where

$$\omega = 1 + f\cos L + g\sin L,\ s^2 = 1 + h^2 + k^2, \text{ and } \eta = m/m_0 \text{ and } \cdot = \frac{d}{dt}.$$

Also, the change in effective velocity is defined as

$$\Delta V_{eff} = -\left(\frac{T}{m_0}\right)\left(\frac{\ln[\eta(t_i)] - \ln[\eta(t_{i-1})}{\eta(t_i) - \eta(t_{i-1})}\right)\Delta t_i \qquad \textbf{(2.19)}$$

While the thrust vector $\overline{T}$ is computed by using two angles α and β, which represent the in-plane and out-plane components of the thrust direction,

$$\bar{T} = \mathrm{T}\begin{Bmatrix} \sin(\alpha)\cos(\beta) \\ \cos(\alpha)\cos(\beta) \\ \sin(\beta) \end{Bmatrix} = \begin{Bmatrix} \square_r \\ \square_\theta \\ \square_h \end{Bmatrix} \quad \textbf{(2.20)}$$

2.3.4 Results

In this study, the scenarios where a spacecraft is transferred from the LEO-to-GEO, LEO-to-MEO, and LEO-to-HEO orbits while the final mass is maximized are analyzed. The conditions for this analysis are illustrated in Table 2 [1].

Table 2 LEO and GEO/MEO/HEO conditions for transfer trajectories [1]

Orbital Element	LEO	GEO	MEO	HEO
Semimajor axis, km	7003	42287	26560	26578
Eccentricity	0	0	0	0.73646
Inclination, deg	28.5	0	54.7	63.435
Right ascension of the ascending node, deg	0	0	0	0
Argument of perigee, deg	0	0	0	0
Mean anomaly, deg	Free	Free	Free	Free

These orbits were specifically chosen since the MEO orbit is representative of a global-positioning-system-type orbit while the HEO orbit portrays the Molniya orbit; in addition, different configurations for the spacecraft are assumed by varying thrust-accelerations (TA) ranging from 10^0 to 10^{-2} N/kg. The different thrust-accelerations considered are 1, 10, 10^{-1}, and 10^{-2} N/kg, which result in a total of 12 orbit transfer cases. Furthermore, in all of these cases, a burn-coast-burn thrusting structure is a priori determined for the transfer trajectories that duplicate the burn structure [1].

Thus, for each of the optimal orbit transfer cases a solution is found using each of the different thrust levels; moreover, the results for the maneuvers are analyzed and all cases are compared on the basis of their effective velocity change, ΔV_{eff}. The results for the LEO-to-GEO, LEO-to-MEO, and LEO-to-HEO transfers are illustrated in Tables 3-5 [1].

Table 3 LEO-to-GEO transfer results [1]

Initial Thrust Acceleration, N/kg	Effective ΔV, m/s First Burn	Second Burn	Total	Total Transfer Time, hours
10^{1}	2366	1761	4127	5.40
10^{0}	2592	1716	4308	6.06
10^{-1}	4079	1088	5167	18.32
10^{-2}	5698	------------	5698	149.59

Table 4 LEO-to-MEO transfer results [1]

Initial Thrust Acceleration, N/kg	Effective ΔV, m/s First Burn	Second Burn	Total	Total Transfer Time, hours
10^{1}	2008	1856	3863	3.06
10^{0}	2137	1834	3970	3.60
10^{-1}	3717	1014	4731	14.56
10^{-2}	5122	------------	5122	135.23

Table 5 LEO-to-HEO transfer results [1]

Initial Thrust Acceleration, N/kg	Effective ΔV, m/s First Burn	Second Burn	Total	Total Transfer Time, hours
10^{1}	2434	836	3271	6.03
10^{0}	2666	890	3555	6.55
10^{-1}	4146	1125	5271	18.59
10^{-2}	6109	------------	6109	159.75

In all of these cases, the highest thrust acceleration, 10 N/kg (approximately 1g), results in a transfer where the burn duration is small compared to the coast arc which indicates that the performance of the transfer is near the performance for a transfer that uses a high-thrust impulsive approximation; a burn duration that is small compared to the coast arc is a characteristic of a transfer based on a high-thrust impulsive approximation [1].

A noticeable aspect of the results for the three transfer types is that as the thrust-acceleration decreases, the ΔV_{eff} and transfer time increases. For instance, a case that results from a thrust-acceleration (TA) of 1 N/kg has a slightly higher ΔV_{eff} and transfer time than a case resulted from a TA of 10 N/kg. Another observation from the results is that for the scenario that uses a TA of 10^{-1} N/kg, the ΔV_{eff} required is 20-25% more for the LEO-to-GEO and LEO-to-MEO cases [1].

For all three-transfer types, the three-dimensional trajectories are illustrated in Figures 3-5 as near optimal, low-thrust transfers taking on the shape of spirals with increasing radius. Here, the thick lines represent a burn arc, and the thin line denotes coast arcs [1].

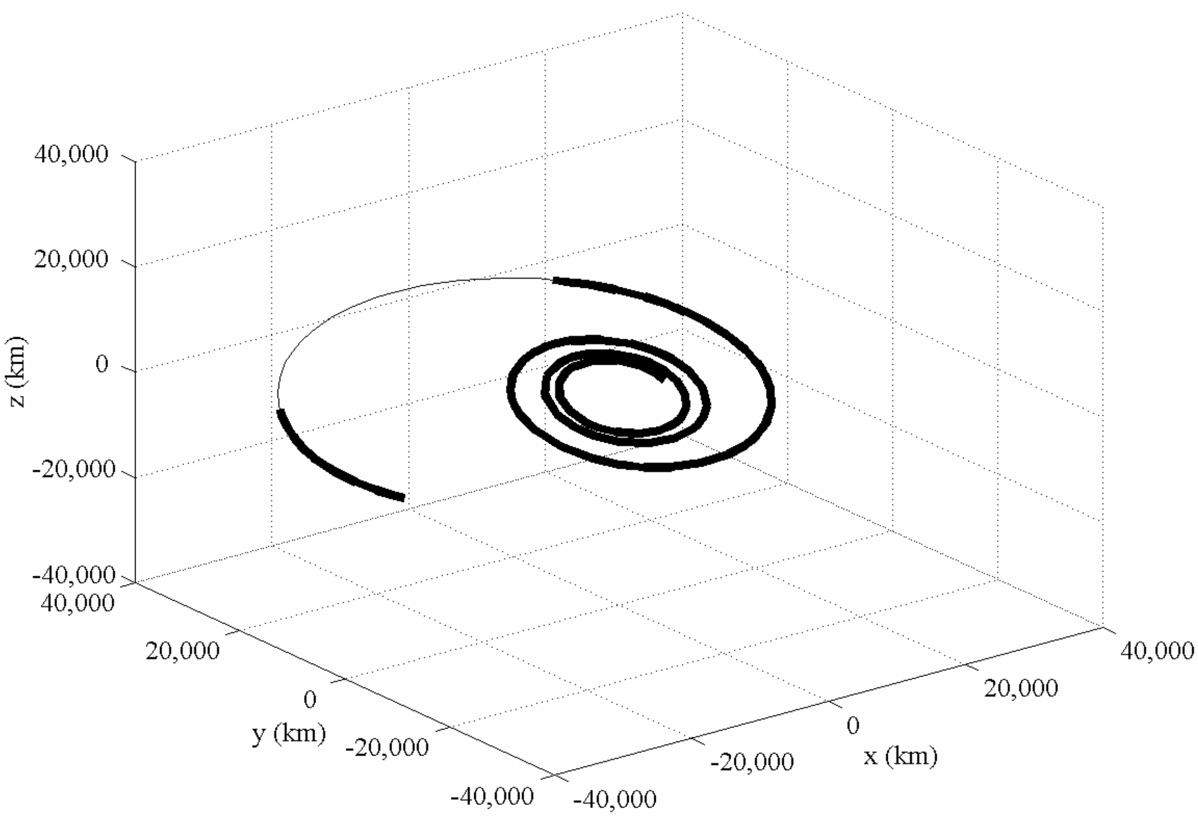

Figure 3: LEO-to-GEO transfer with initial thrust acceleration of 10^{-1} N/kg [1]

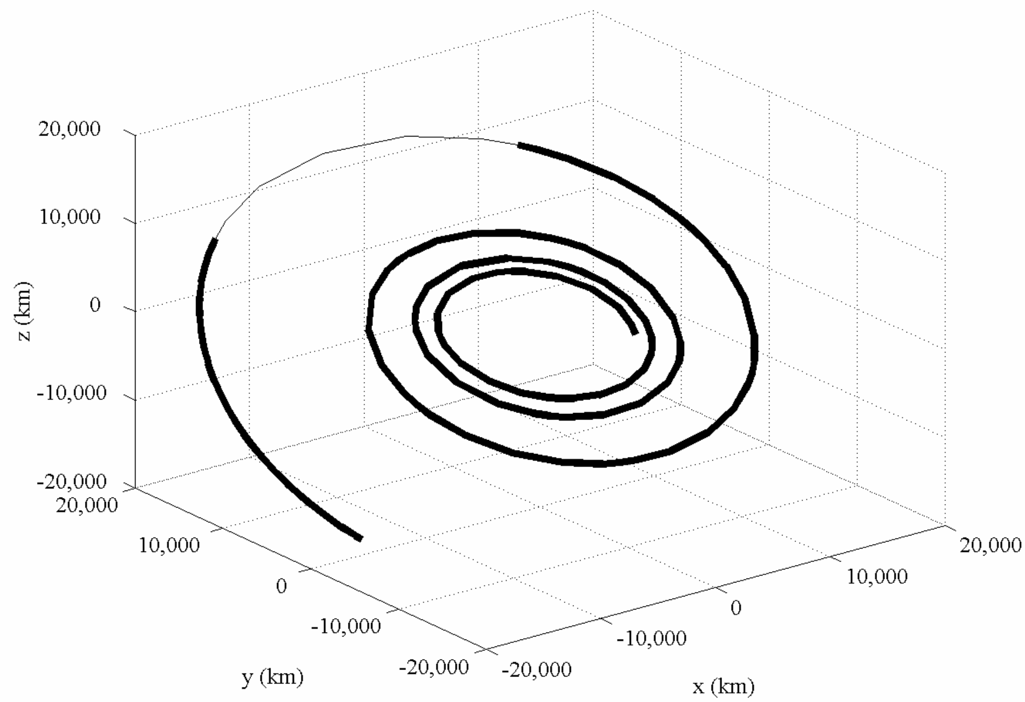

Figure 4: LEO-to-MEO transfer with initial thrust acceleration of 10^{-1} N/kg [1]

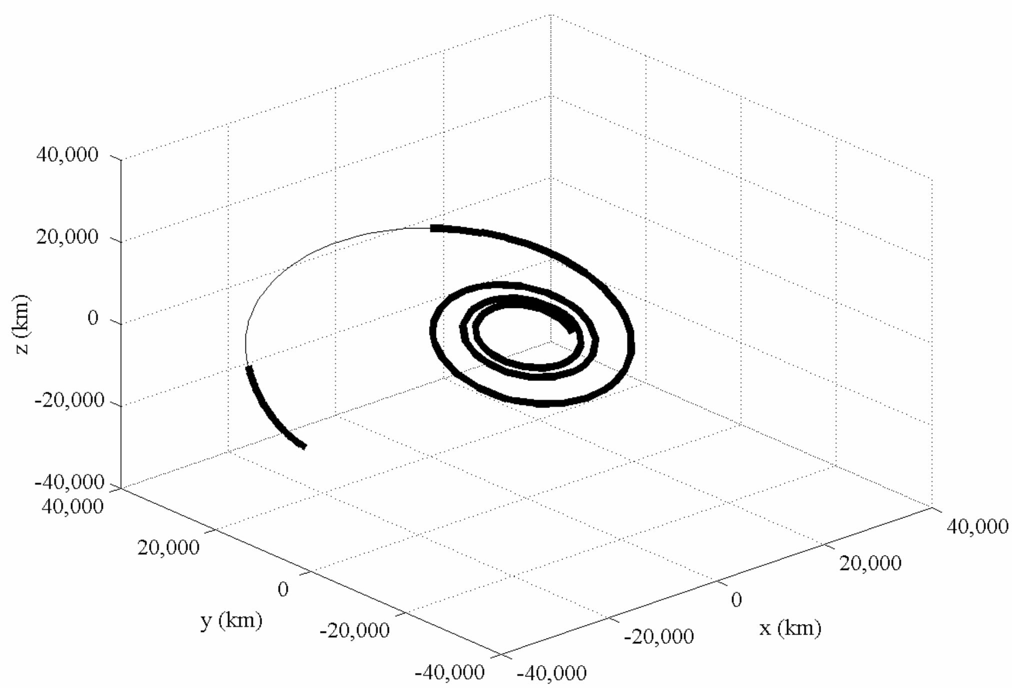

Figure 5: LEO-to-HEO transfer with initial thrust acceleration of 10^{-1} N/kg [1]

The ΔV_{eff} for the LEO-to-GEO and LEO-to-MEO transfer type are approximately 35% greater than the corresponding transfer types with a TA of 10 N/kg, and the LEO-to-HEO transfer type is approximately 85% more than the corresponding transfer with a TA of 10 N/kg [1].

2.4 SEPSPOT

SEPSPOT is a modified version of the SECKSPOT (Solar Electric Control Knob Setting Program by Optimal Trajectories) computer program. The program is written in Fortran IV with double precision. A costate formulation is used which results in a two point boundary value problem which is solved using a Newton iteration on the initial unknown parameters and the unknown transfer time. Also, a Runge-Kutta method is used to integrate the state and costate equations and averaging is done using a Gaussian quadrature [16, 17].

SEPSPOT is designed to calculate time optimal or nearly time optimal geocentric transfers for a solar electric spacecraft with or without attitude constraints. The program has the option to use initial high thrust or low thrust. For the initial high thrust stage one or two impulses of fixed total ΔV can be included, and the initial orbit is assumed to be circular. For the low thrust stage, a nonsingular set of orbital elements and an averaging method are used. In addition, the low thrust phase is applicable to general geocentric elliptical orbits [16,17].

The program also includes options for oblateness, solar motion, shadowing with or without delay in thruster startup, and an analytic radiation and power degradation model. The main modifications done to the original SECKSPOT program include the altitude constraint solution, a new radiation and power loss model, a revised shadow model, and extended output. Also, one key aspect of using the altitude constraints option is that it causes power to become a function of thrust direction and sun direction, and the time optimal thrust direction becomes a complex function of primer vector direction [16,17].

SEPSPOT's input is entered in the form of a data file (input file), which contains initial values of unspecified states and costates and a guess for the transfer time. The initial and desired orbit are specified in terms of semimajor axis (km), eccentricity, angle of inclination (degrees), longitude of ascending node (degrees), and argument of perigee (degrees). The initial thrust acceleration must be included in terms of initial mass (kg), initial power (kw), specific impulse (sec), and total constant efficiency (ε) [16, 17].

2.4.1 SEPSPOT's Equations

SEPSPOT operates on a set of equinoctial elements. These equations [17] are shown in Eqs. 2.21-2.30:

a) Equinoctial Orbital Elements (in terms of classical elements):

$$a = a \tag{2.21}$$

$$h = e \sin(\omega + \Omega) \tag{2.22}$$

$$k = e \cos(\omega + \Omega) \tag{2.23}$$

$$p = \tan(i/2) \sin \Omega \tag{2.24}$$

$$q = \tan(i/2) \cos\Omega \tag{2.25}$$

b) Inverse Relationships:

$$a = a \tag{2.26}$$

$$e = \sqrt{h^2 + k^2} \tag{2.27}$$

$$i = 2\tan^{-1}\sqrt{p^2 + q^2} \tag{2.28}$$

$$\Omega = \tan^{-1}(p/q) \tag{2.29}$$

$$\omega = \tan^{-1}(h/k) - \tan^{-1}(p/q) \tag{2.30}$$

The costate equations [17], used for the optimization, are shown in Eqs. 2.31-2.40:

a) In terms of Equinoctial Orbital Elements (λ and ψ = adjoints):

$$\lambda_a = \psi_a \tag{2.31}$$

$$\lambda_h = \psi_e \sin(\omega + \Omega) + \psi_\omega \cos(\omega + \Omega)/e \tag{2.32}$$

$$\lambda_k = \psi_e \cos(\omega + \Omega) - \psi_\omega \sin(\omega + \Omega)/e \tag{2.33}$$

$$\lambda_p = \psi_i \, 2\sin\Omega \, \cos2(i/2) + \psi_\Omega \cos\Omega/\tan(i/2) - \psi_\omega \cos\Omega/\tan(i/2) \qquad \textbf{(2.34)}$$

$$\lambda_q = \psi_i \, 2\cos\Omega \, \cos2(i/2) - \psi_\Omega \sin\Omega/\tan(i/2) + \psi_\omega \sin\Omega/\tan(i/2) \qquad \textbf{(2.35)}$$

b) In terms of Classical Orbital Elements:

$$\lambda_a = 0 \qquad \textbf{(2.36)}$$

$$\lambda_h = \psi_e \frac{h}{\sqrt{h^2+k^2}} + \psi_\omega \frac{k}{h^2+k^2} \qquad \textbf{(2.37)}$$

$$\lambda_k = \psi_e \frac{k}{\sqrt{h^2+k^2}} - \psi_\omega \frac{h}{h^2+k^2} \qquad \textbf{(2.38)}$$

$$\lambda_p = \psi_i \frac{2p}{\sqrt{p^2+q^2}} (1 + p^2 + q^2) + \psi_\Omega \frac{q}{p^2+q^2} - \psi_\omega \frac{q}{p^2+q^2} \qquad \textbf{(2.39)}$$

$$\lambda_q = \psi_i \frac{2q}{\sqrt{p^2+q^2}} (1 + p^2 + q^2) - \psi_\Omega \frac{p}{p^2+q^2} + \psi_\omega \frac{p}{p^2+q^2} \qquad \textbf{(2.40)}$$

Chapter 3

ANALYSIS

3.1 Preliminary Work

A zip file containing the SEPSPOT program files and the program's manual and analysis in the form of a pdf file were obtained from NASA/Glenn Research Center. The zip file was uncompressed and Jeff Nucciarone, the Pennsylvania State University senior research programmer for the High Performance Computing Group, ITS/ASET, compiled the program with the Linux operating system. After the program was compiled a cluster was opened, LIONXL, in Pennsylvania State University's High Performance Computing Center and all the program files were copied into a folder named "sepspot."

To run and execute SEPSPOT a program called "SSH Secure Shell" was used to connect via the Internet to the LIONXL cluster, which uses the Linux system. The SSH Secure Shell program was installed and ran in both Intel Celeron (533 MHz with 319 RAM) and Intel Pentium III (930 MHz with 256 RAM) computers with Windows ME and Windows XP operating systems.

3.2 Variables: Independent and Dependent

Tables 6 and 7 show the initial conditions required to run the computer simulation. The initial conditions used to declare the initial and final orbit consist of the semimajor axis (km), eccentricity, inclination (degrees), right ascension of the ascending node (degrees), and argument of perigee (degrees) for both the LEO and Molniya orbits. In addition, the initial mass (kg), initial power (kw), thruster specific impulse (sec), and the total transfer time (hours) are considered in order to compute the initial thrust acceleration using the following relation [16,17,18]:

$$P = \frac{g_0 T I_s}{2\varepsilon} \qquad \textbf{(3.1)}$$

Also, the final conditions are the total effective change in velocity, the total transfer time (hours), the semimajor axis time history, the eccentricity time history, the inclination time history, the apogee and perigee radius time history, and the energy time history.

Table 6 Initial Conditions for the Initial and Final Orbit

Element	Initial Value (LEO)	Final Value (Molniya)
a	7000 km	26578 km
e	0	0.73646
i	28.5°	63.435°
Ω	0°	0°
ω	0°	0°

Table 7 Initial Conditions for Initial Thrust Acceleration

Element	TA = 10^{-1} N/kg	TA = 10^{-2} N/kg
Initial Mass (kg)	1	1
Initial Power (kw)	0.4905	0.04905
ISP (sec)	1000	1000
Estimated Time of Arrival (hours)	13.5	139.2

For this study the initial conditions are regarded as the independent variables since all other data is derived from these conditions. The final conditions are considered to be the dependent variables since they are derived from the initial conditions or the independent variables. Figure 6 depicts the orientation of the right ascension of the ascending node (Ω), the argument of perigee (ω), and the inclination (i).

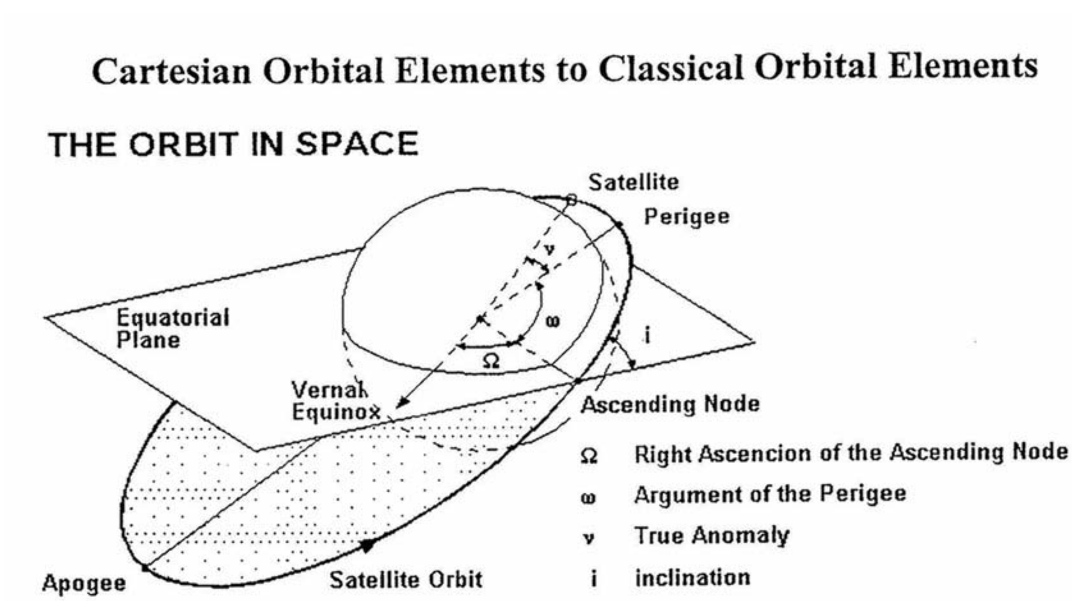

Figure 6: Orbit elements

3.3 Instrumentation

A quantitative numerical research method is used in this study. The main instrument used during this project is the computer program, SEPSPOT, since it collects the data by computing the final conditions from the initial conditions. This instrument can be considered reliable since it will always be consistent with its solutions; the program is based on mathematical equations.

Furthermore, another instrument that is used in the study is the analytical data that was produced during the original project since it serves as secondary data. The analytical solutions obtained from Spencer's research study serves as a tool to validate the data produced by SEPSPOT.

3.4 Data Collection and Analysis

For this project, the final conditions are derived from the solutions produced by SEPSPOT. In addition, the final data is analyzed by using Microsoft Excel to plot and compare the results with Spencer's analytical solutions. Overall, there are a total of five plots for each initial thrust acceleration (10^{-1} N/kg and 10^{-2} N/kg). For these graphs, the semimajor axis (km), eccentricity, inclination (degrees), apogee and perigee radius (km), and energy are all plotted versus time (hours).

Chapter 4

RESULTS

4.1 LEO-Molniya Transfer: $T/m_0 = 10^{-1}$ N/kg

For the case of an initial thrust acceleration of 10^{-1} N/kg, SEPSPOT accomplishes the transfer in one burn while achieving an overall effective change in velocity (ΔV) of 5814.69 m/s in 12.18 hours. On the other hand, Spencer's analytical solution completes the transfer using two burns in 13.75 hours at a ΔV of 6896.00 m/s. Refer to Table 8 to view a comparison of the two solutions, which shows that SEPSPOT's trajectory is slightly more efficient by completing the transfer in less time; in addition, there is a percent error of 12.89% between the numerical solution provided by SEPSPOT and Spencer's analytical solution.

Table 8 Initial Thrust Acceleration of 10^{-1} N/kg

	Overall Effective Change in Velocity (ΔV)	Overall Time	Percent Error (%)
Spencer's Results	6896.00 m/s	13.75 hrs	12.89%
SEPSPOT's Results	5814.69 m/s	12.18 hrs	

Five figures are now presented for this case. Figures 7-11 show a comparison of the time history of the semimajor axis, eccentricity, inclination, apogee and perigee radius, and energy between SEPSPOT's numerical data and Spencer's analytical data. A key aspect of the comparison that should be noticed is that SEPSPOT's trajectory manages to complete all the desired conditions in approximately the amount of time it takes Spencer's trajectory to complete the first burn.

Figure 7 shows how SEPSPOT manages to achieve a semimajor axis of 26578 km (Molniya Orbit) from a starting semimajor axis of 7000 km (LEO Orbit) in 12.18 hours by using one burn. However, Spencer's trajectory shows that a burn is performed for approximately 12 hours, followed by a coast arc of 4.5 hours, and then a second burn is made which takes about 1.5 hours to complete the trajectory. One should also notice that SEPSPOT's trajectory seems almost parabolic while Spencer's trajectory has more of a oscillatory shape which could account for SEPSPOT's reduced time to complete the trajectory.

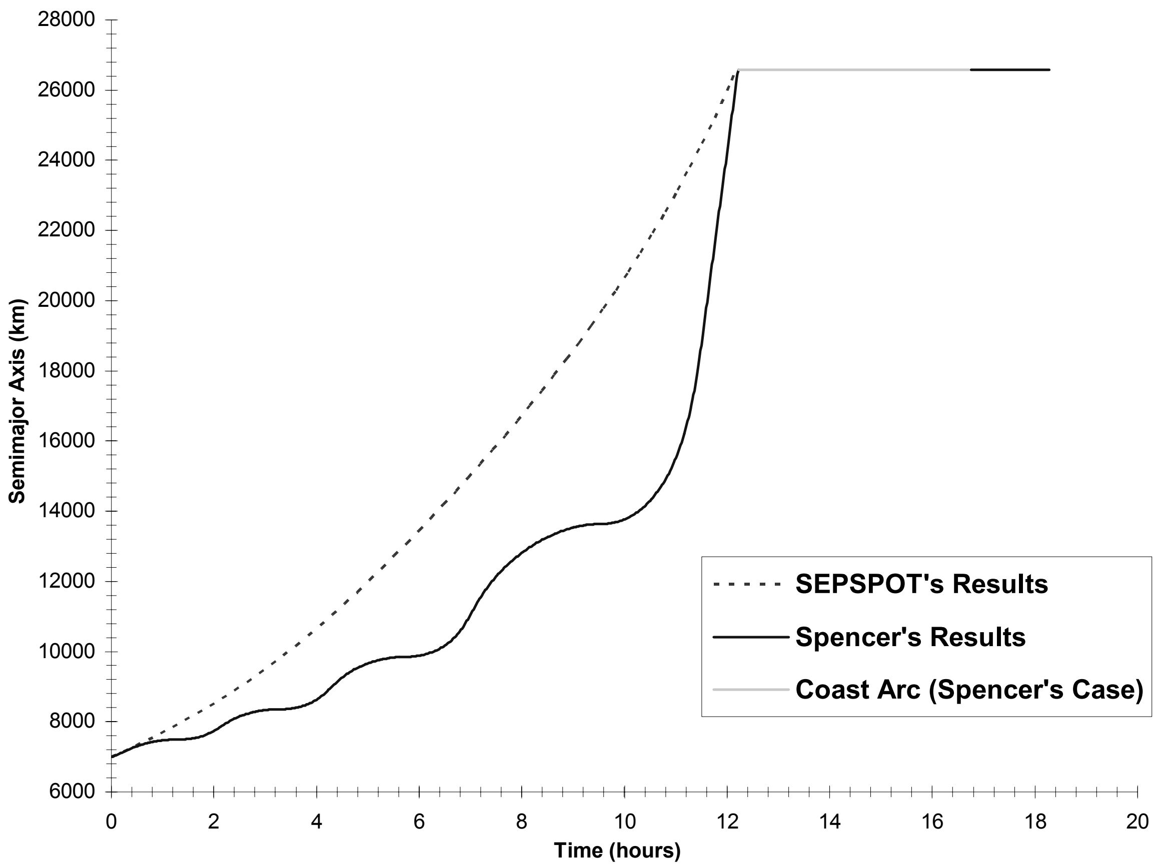

Figure 7: Semimajor Axis Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-1}$

Figure 8 shows how Spencer's eccentricity curve slightly oscillates, but increases at a steady rate while SEPSPOT's curve increases slowly at the beginning and then the eccentricity starts increasing at a faster rate, resulting in a smooth parabolic curve. The oscillations in Spencer's trajectory could account for additional time required to achieve a final eccentricity of 0.73646.

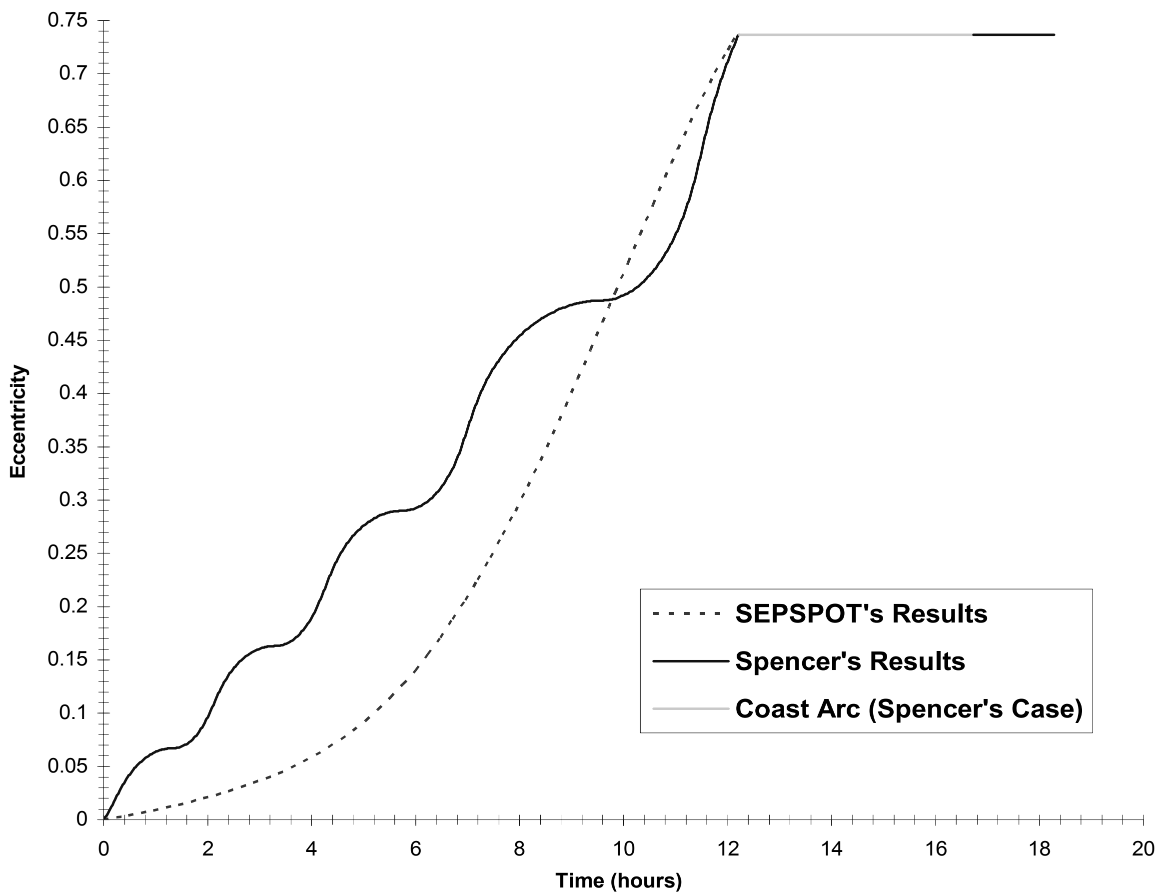

Figure 8: Eccentricity Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-1}$

Figure 9 illustrates how for Spencer's results the angle of inclination stays constant at 28.5° during the first burn and then it rapidly increases to 63.435° in the second burn. SEPSPOT's results show how the angle of inclination is increased over time and a parabolic curve is formed.

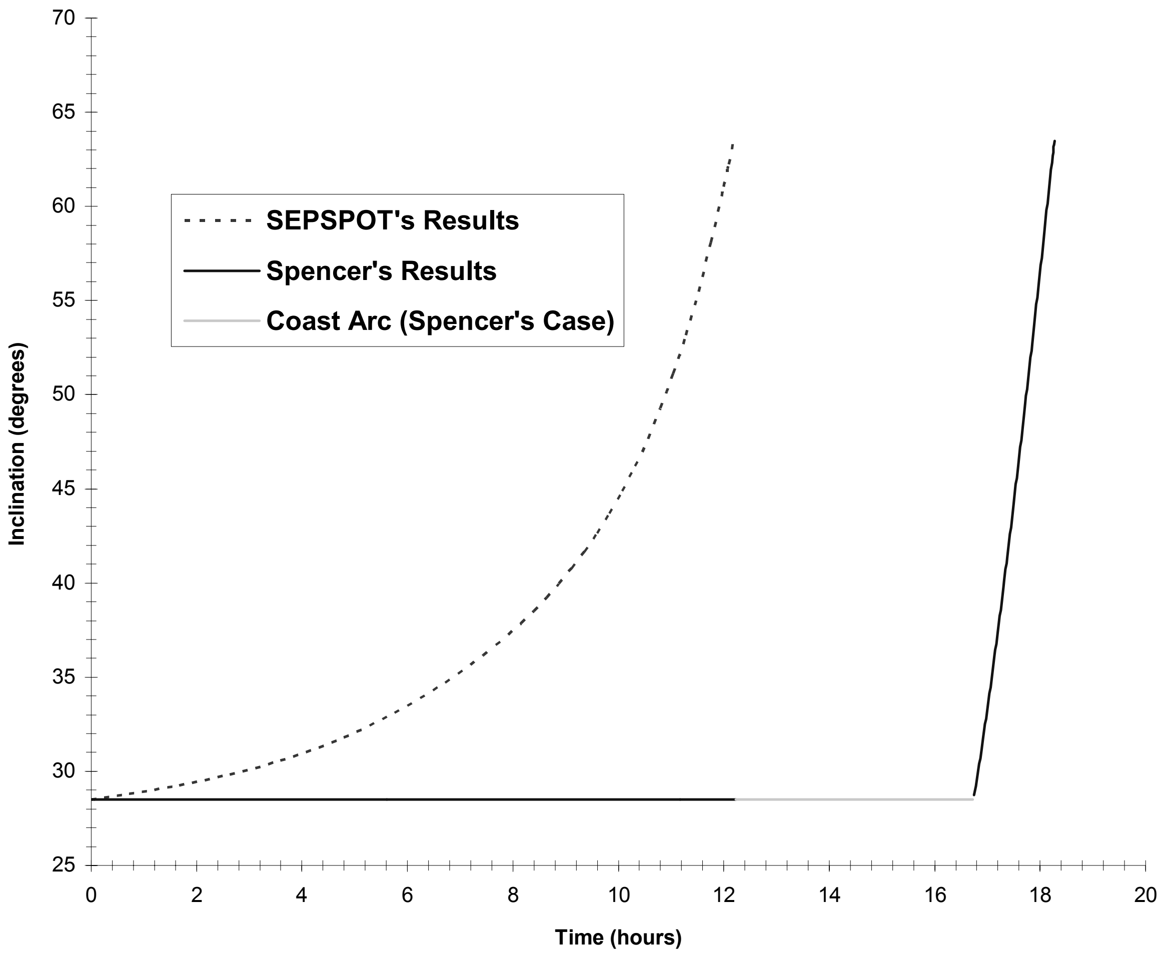

Figure 9: Inclination Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-1}$

Figure 10 shows how SEPSPOT's trajectory achieves both the apogee and perigee radius desired conditions in the amount of time it takes Spencer's trajectory to complete the first burn. An important aspect of this plot is that SEPSPOT keeps the perigee radius free which forms a parabolic type curve while Spencer maintains a constant perigee radius. This is a key difference since it might account for the optimal trajectory found by SEPSPOT.

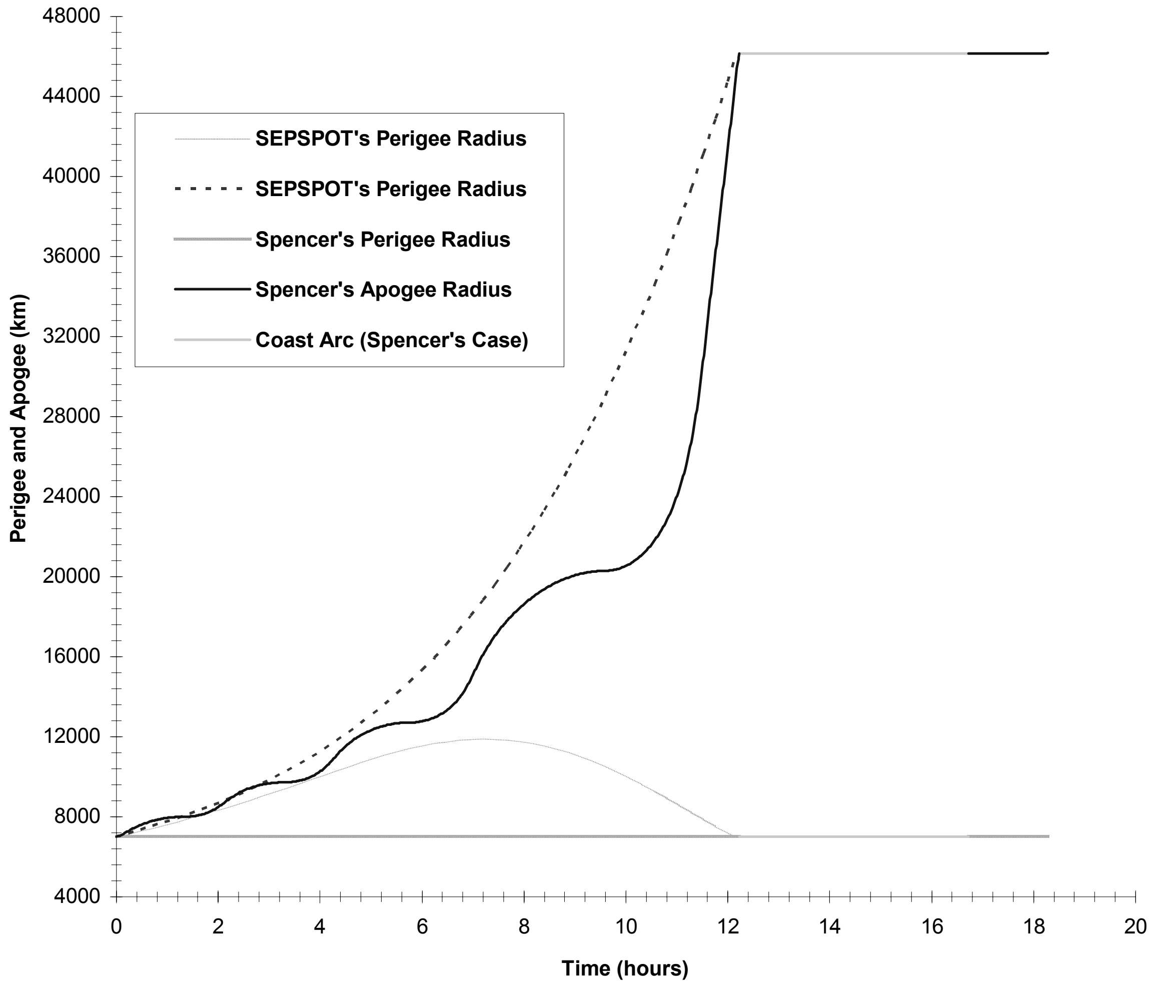

Figure 10: Apogee and Perigee Radius Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-1}$

From Figure 11 one can see that SEPSPOT's trajectory achieves the energy levels required to complete the trajectory in less time. In addition, from the plot Spencer's data indicates that more energy is required to accomplish the desired LEO-Molniya transfer.

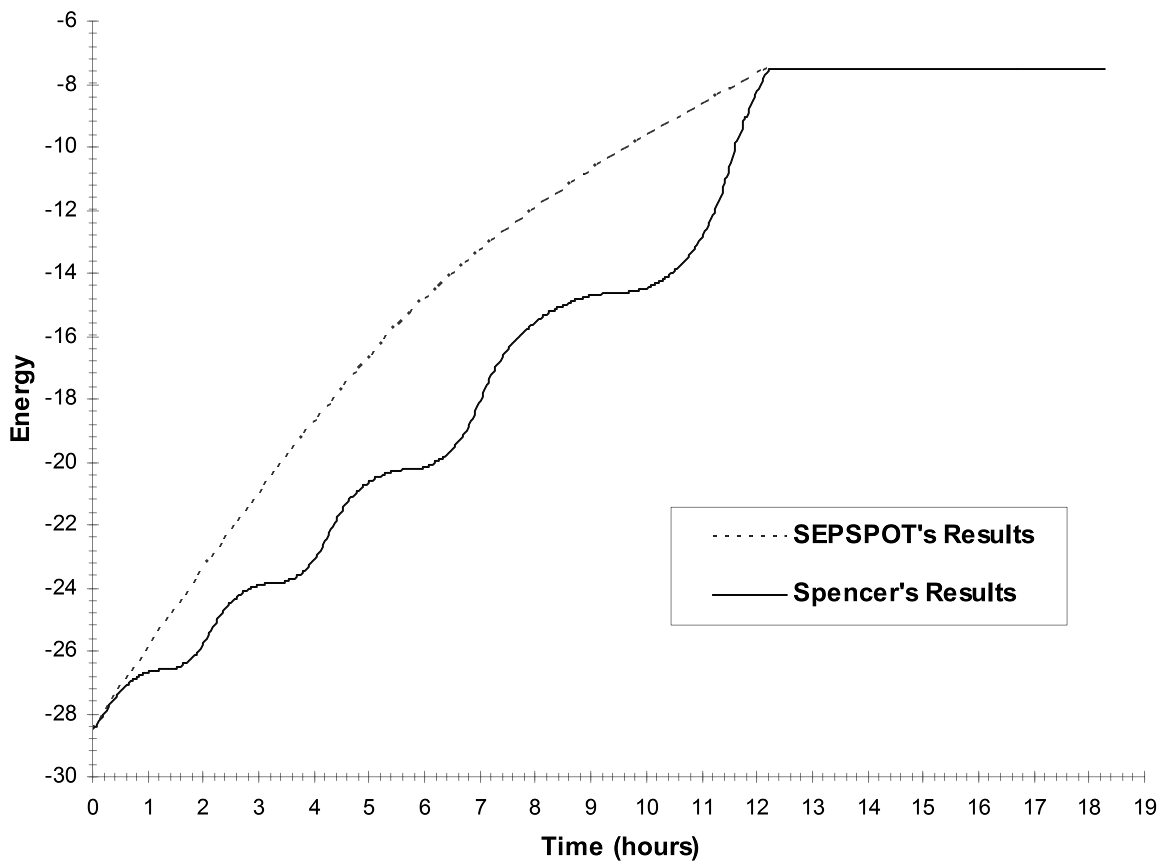

Figure 11: Energy Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-1}$

4.2 LEO-Molniya Transfer: $T/m_0 = 10^{-2}$ N/kg

In the case of an initial thrust acceleration of 10^{-2} N/kg, SEPSPOT completes the transfer in one burn while achieving a ΔV of 5814.69 m/s in 121.77 hours. Spencer's analytical solution achieves the transfer using one burn in 136.83 hours at a ΔV of 6844.00 m/s. Table 9 depicts a comparison of the two solutions in which SEPSPOT's trajectory is shown to be slightly more optimal by completing the transfer in less time. Furthermore, there is a percent error of 12.37% between the numerical solution provided by SEPSPOT and Spencer's analytical solution.

Table 9 Initial Thrust Acceleration of 10^{-2} N/kg

	Overall Effective Change in Velocity (ΔV)	Overall Time	Percent Error (%)
Spencer's Results	**6844.00 m/s**	**136.83 hrs**	**12.37%**
SEPSPOT's Results	**5814.69 m/s**	**121.77 hrs**	

Figures 12-16 illustrate a comparison of the time history of the semimajor axis, eccentricity, inclination, apogee and perigee radius, and energy between SEPSPOT's numerical data and Spencer's analytical data. One should note that unlike the previous case, both SEPSPOT's and Spencer's trajectories manage to achieve the desired orbit using only one burn.

Figure 12 shows that despite the fact that both trajectories only require one burn to complete the desired final conditions, Spencer's results are still taking longer to reach the final conditions. In this case SEPSPOT's semimajor axis curve is smooth and parabolic while Spencer's curve has a similar shape, but it contains small oscillations; these oscillations can account for the additional time required to reach the Molniya orbit.

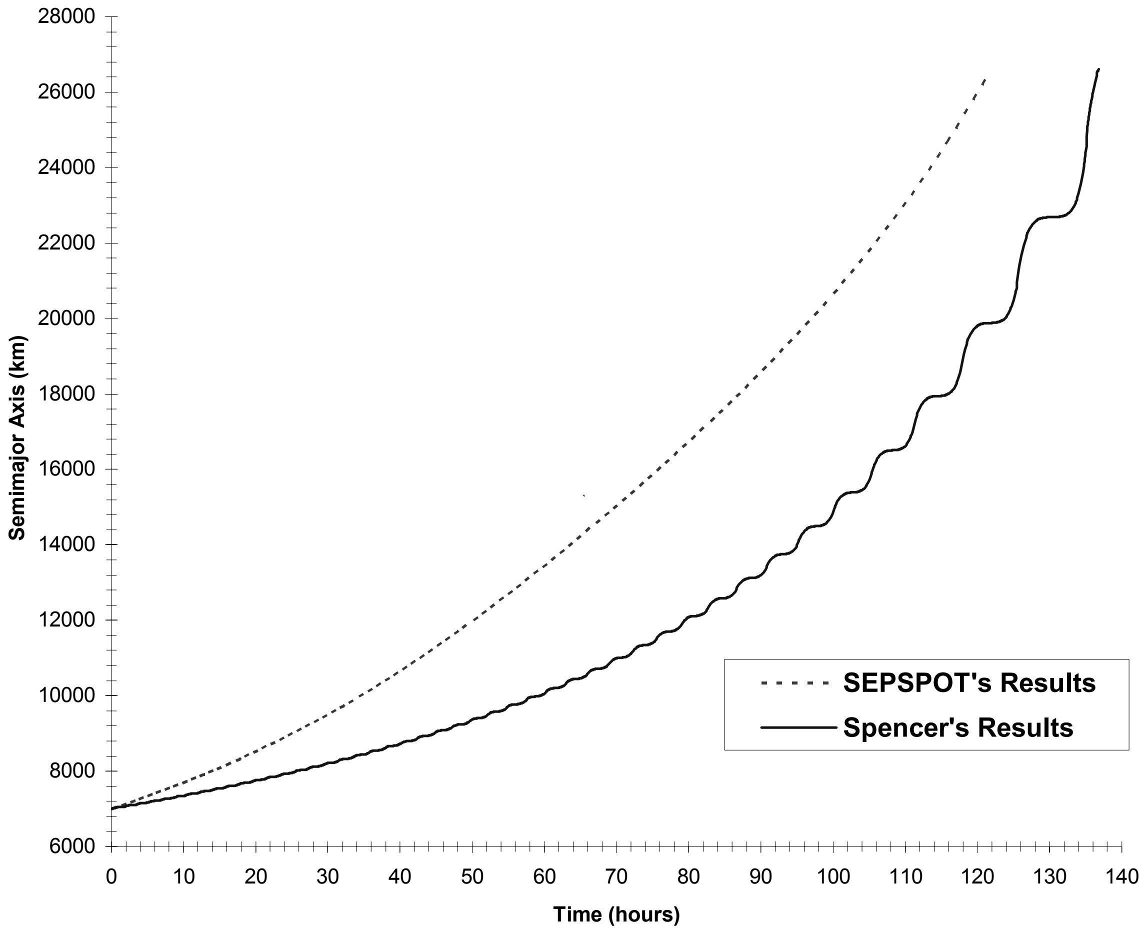

Figure 12: Semimajor Axis Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-2}$

Figure 13 shows how for Spencer's results the eccentricity seems to increase at an almost constant rate with time. Furthermore, for SEPSPOT's results, like in previous initial thrust acceleration case, the curve increases slowly at the beginning and then the eccentricity starts increasing at a faster rate, resulting in a smooth parabolic curve.

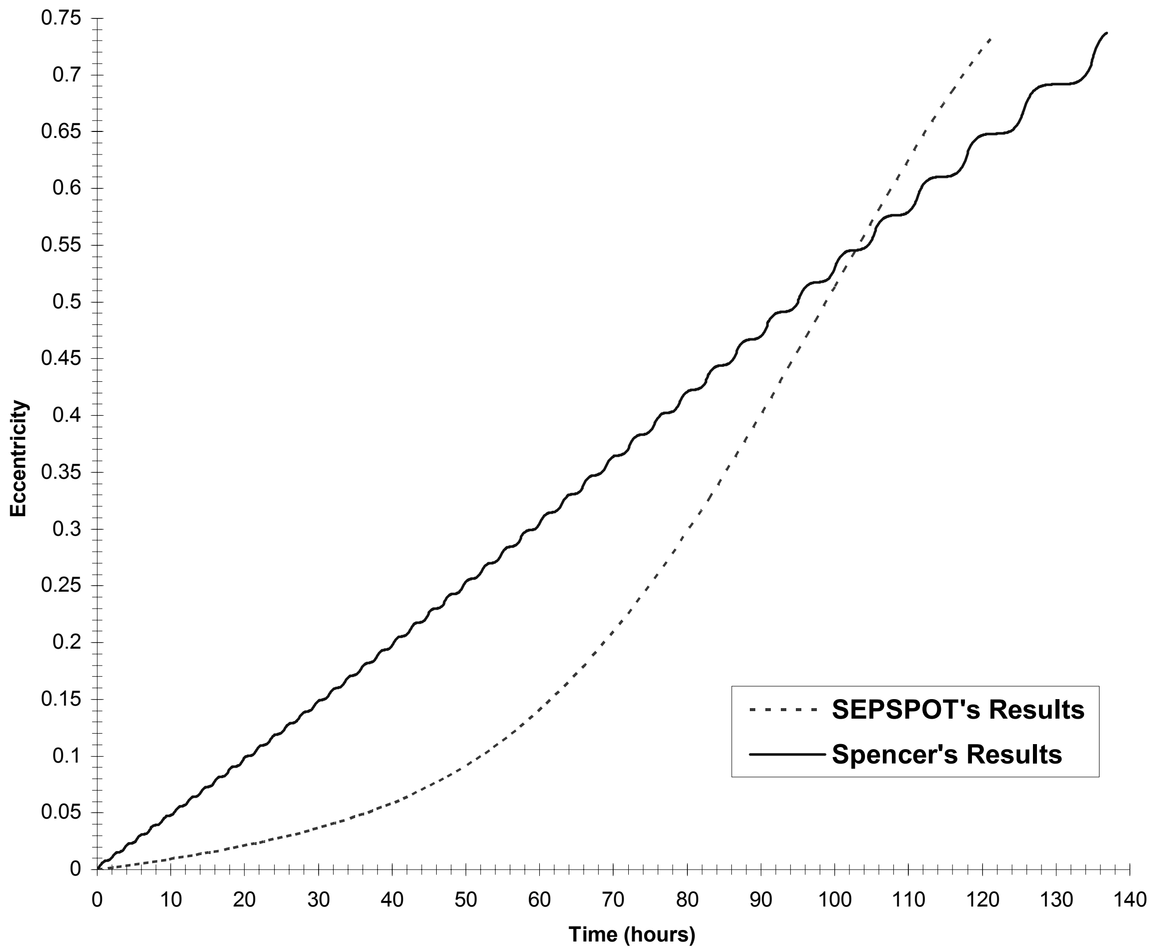

Figure 13: Eccentricity Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-2}$

Figure 14 illustrates how for both Spencer's and SEPSPOT's results, the angle of inclination is increased over time and a parabolic curve is formed. The only difference is that SEPSPOT's curve is smooth while Spencer's curve has oscillations. In addition, the plot depicts how the angle of inclination for SEPSPOT's trajectory increases at a much faster rate than Spencer's trajectory, hence reaching the desired angle of inclination in less time.

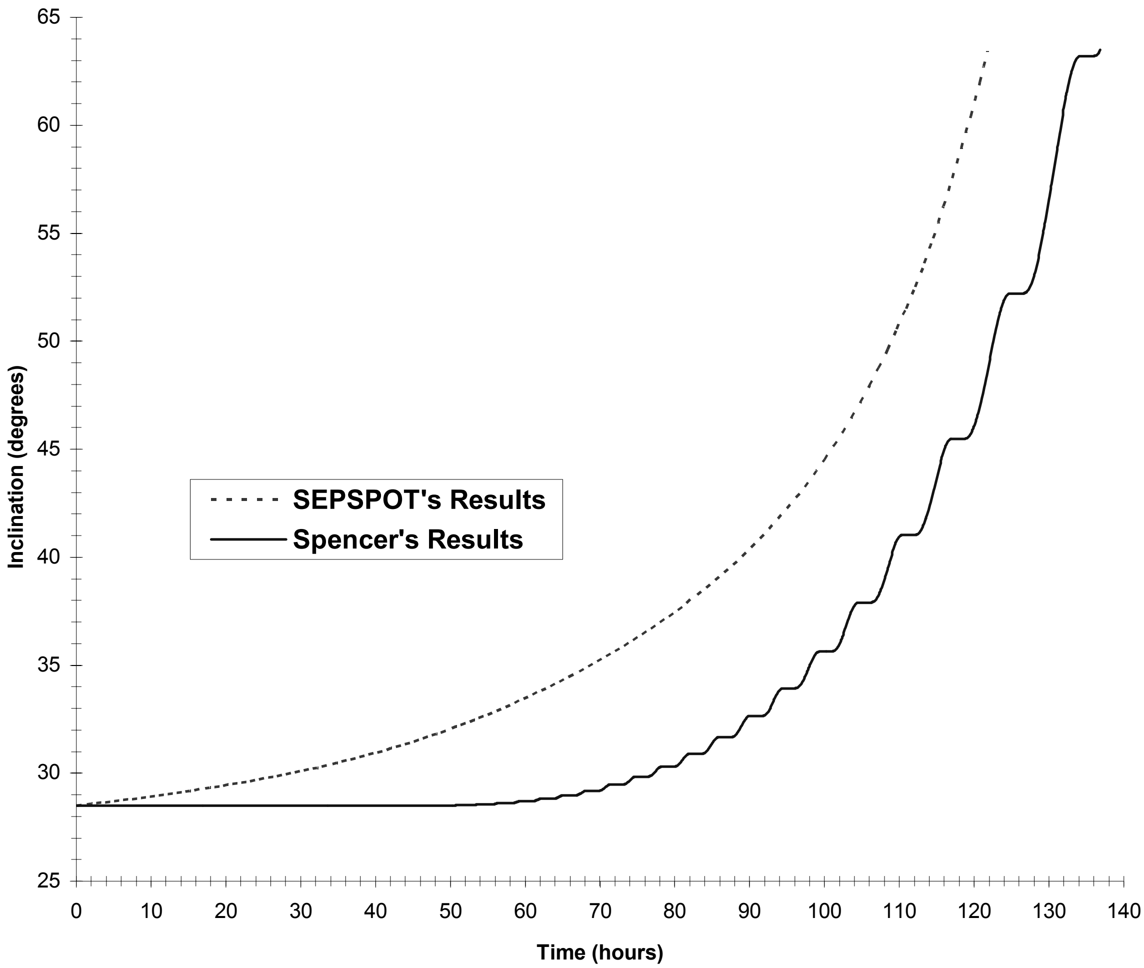

Figure 14: Inclination Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-2}$

Figure 15 illustrates how SEPSPOT keeps the perigee radius free and forms a parabolic type curve while Spencer maintains a constant perigee radius. This is a key difference since it might account for SEPSPOT's more efficient trajectory.

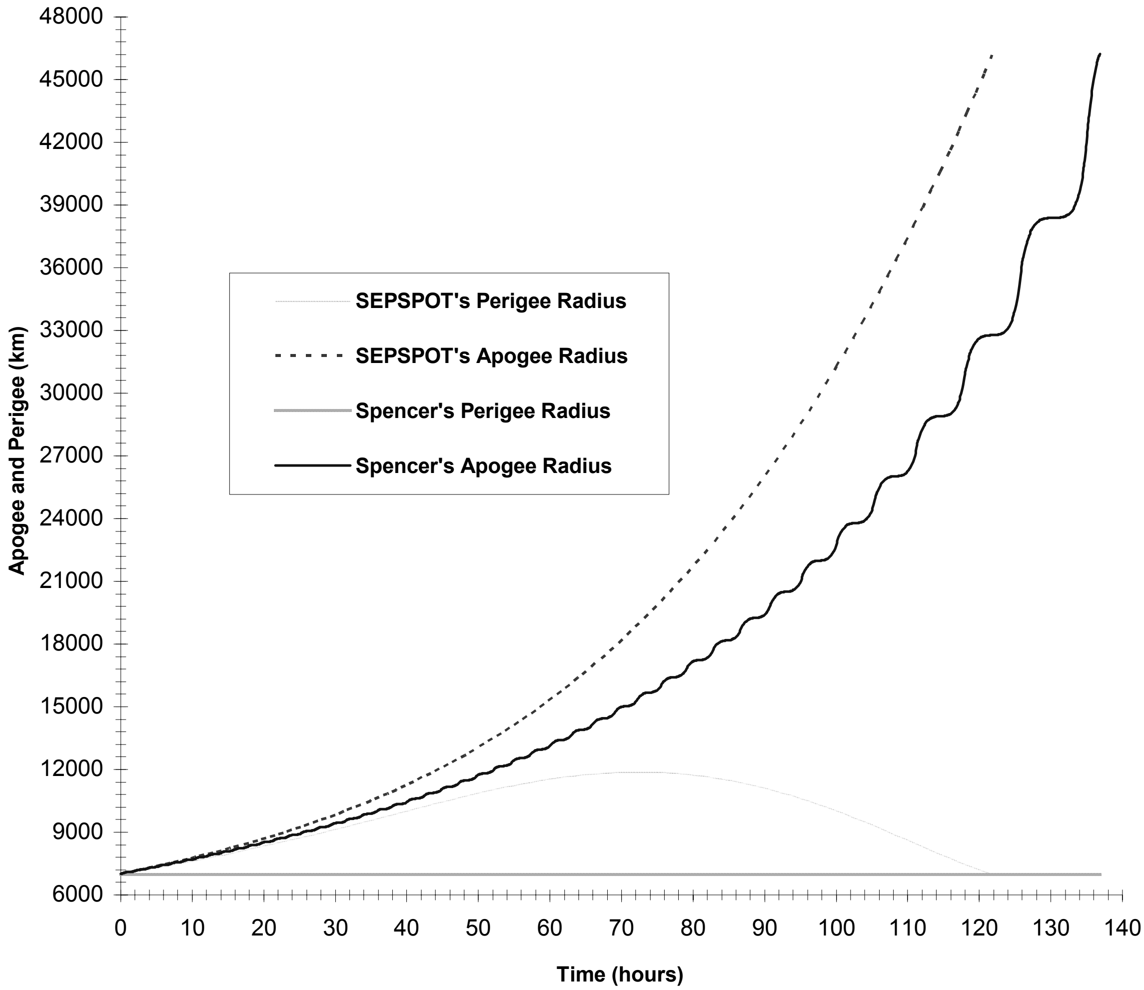

Figure 15: Apogee and Perigee Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-2}$

Figure 16 shows how SEPSPOT's trajectory achieves the energy levels required to complete the trajectory in less time. In addition, from the plot Spencer's data indicates that more energy is required to accomplish the desired LEO-Molniya transfer.

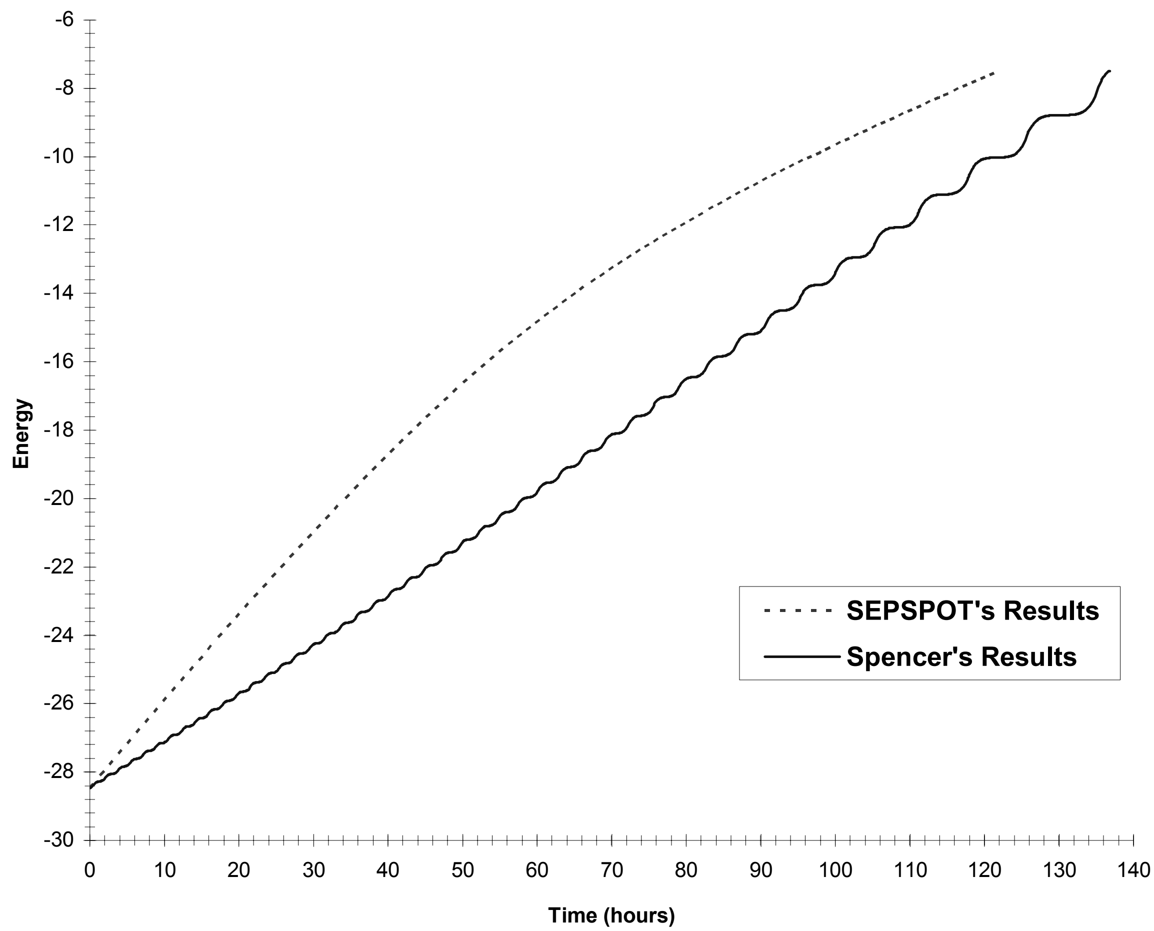

Figure 16: Energy Time History for Three-Dimensional, LEO-Molniya Transfer, $T/m_0 = 10^{-2}$

Chapter 5

CONCLUSIONS

5.1 SEPSPOT Evaluation

SEPSPOT proved to be a very useful and effective tool for the evaluation of Spencer's analytical data. However, one disadvantage is that in order to be able to run the program the user must have knowledge of the costates and of the total transfer time. If the values inputted into SEPSPOT are not close to the actual values then the program will not converge or it will converge to the incorrect solution. This means that SEPSPOT is only useful for evaluating transfers where the user has an idea of how long the transfer should take.

Also, another disadvantage is that the program does not have a graphical interface and the data is inputted and outputted in the form of a data file. SEPSPOT is very particular about the format of the input data, and if the format is incorrect the program will give an error message or it will run and output useless data files (empty).

Despite these disadvantages, one great advantage is that there is a great amount of flexibility in running SEPSPOT since the SSH Secure Shell Program can be run from any Windows based operating system that has internet capabilities. This allows the user to run and execute SEPSPOT from any Windows PC whether it's in a computer lab or the users own home.

5.2 Spencer's Analytical Data Evaluation

A comparison between Spencer's analytical data and SEPSPOT's numerical data showed that the percent error between the two is very small, there is about a 13% percent difference for the initial thrust acceleration of 10^{-1} N/kg and 10^{-2} N/kg. The factor that affected the results of both solutions is that for Spencer's analytical solution the radius of perigee was held constant while with SEPSPOT's numerical solution the radius of perigee was free. This difference indicates that while Spencer's solution was closely related to SEPSPOT's solution it can be improved by doing another analytical analysis with the radius of perigee free.

5.3 Future Work

During this study, only two cases were evaluated using SEPSPOT (TA = 10^{-1} N/kg and TA=10^{-2} N/kg). The cases for initial thrust acceleration of 10^{0}, 10^{1}, 10^{2}, 10^{3}, 10^{4}, and 10^{5} N/kg will be run in SEPSPOT at a later time to evaluate Spencer's analytical solutions for these cases. After evaluating all these cases another project will be conducted in which new analytical solutions will be computed for the LEO-to-Molniya transfer where the radius of perigee will be free instead of constant.

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